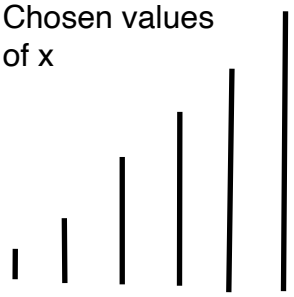
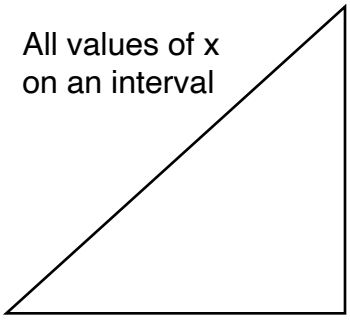
To take the integral of a function is to find the area or volume of a one higher dimensional object whose cross sections correspond to the value of the function at a given point:

Integral of x:

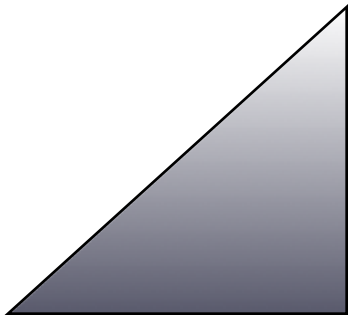
Chosen values of x

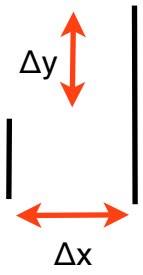


All values of x on an interval



The integral of x on this interval will yield the area of the figure





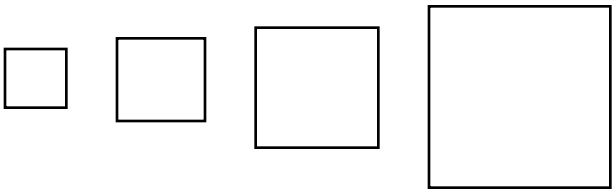
Notice that, in this case, $\Delta y = \Delta x$ which means the rate of change is given by:

$\frac{\Delta y}{\Delta x} = \frac{\Delta x}{\Delta x}$ or

$\frac{\Delta y}{\Delta x} = 1$

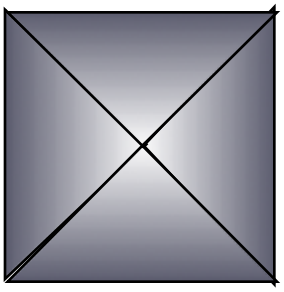
Integral of x^2:

Chosen values of x^2

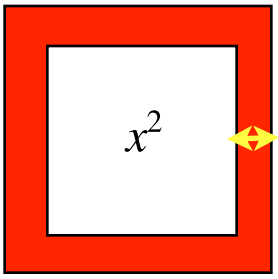


In this case, we get a three dimensional object with square cross sections

All values of x^2 on an interval (viewed from above)



We get a square pyramid.



$$(x+\Delta x)^2$$

Notice that, in this case:

$$\Delta y=(x^2+2\Delta x+\Delta x^2)-(x^2)$$

$$\Delta y=2\Delta x+\Delta x^2$$

Which means the rate of change is given by:

$$\frac{\Delta y}{\Delta x}=\frac{(2x\Delta x+\Delta x^2)}{\Delta x}$$

or

$$\frac{\Delta y}{\Delta x}=2x+\Delta x$$